CONTENTS

CHAPTER 1 SET THEORY 1

1.1 SETS 1
1.1.1 Basics of Sets..... 1
1.1.2 Operations on Sets..... 3
1.1.3 Relations..... 7
1.1.4 Functions..... 12
1.1.5 Algebra of Sets..... 15

1.2 THE REAL NUMBER SET..... 20
1.2.1 Introduction..... 20
1.2.2 The real number Set..... 27
1.2.3 The extended real number Set..... 35
1.2.4 Some useful tools..... 37
1.2.5 Representation of a real number..... 41

1.3 THE COMPLEX NUMBER SET..... 43
1.3.1 Introduction..... 43
1.3.2 Algebraic form of the complex numbers..... 45
1.3.3 Trigonometric form of the complex numbers..... 46

CHAPTER 2 LINEAR EQUATION SYSTEMS..... 50

2.1 MATRICES..... 50
2.1.1 Vector spaces..... 50
2.1.2 Basics of matrices..... 53
2.1.3 Operations on matrices..... 58

2.2 DETERMINANTS..... 62
2.2.1 Combinatorial analysis..... 62
2.2.2 Basics of determinants..... 67
2.2.3 Properties of determinants 70
2.2.4 Minor determinants..... 74

2.3 LINEAR EQUATION SYSTEMS..... 89
2.3.1 Introduction..... 89
2.3.2 Solution of linear equation systems 91
2.3.3 Elimination procedure of Gauss..... 97

2.4 THE EIGENVALUE PROBLEM..... 100
2.4.1 Eigenvalues and eigenvectors..... 100

CHAPTER 3 BASIC TOPOLOGY..... 104

3.1 METRIC SPACES..... 104
3.1.1 Introduction..... 104
3.1.2 The metric space..... 106
3.1.3 Open set. Neighborhood of a point..... 107
3.1.4 Accumulation point. Closed set..... 110
3.1.5 Compact set..... 113

3.2 THE METRIC SPACE $\mathbb{R}^n$..... 115
3.2.1 Case $n = 1$..... 115
3.2.2 Case $n > 1$..... 118

CHAPTER 4 NUMERICAL SEQUENCES AND SERIES..... 123

4.1 SEQUENCES OF REAL NUMBERS..... 123
4.1.1 Regular sequences..... 123
4.1.2 Operations on limits 135
4.1.3 Upper and lower limit..... 146
4.1.4 Cauchy sequences..... 152
4.1.5 The number e..... 154

4.2 SERIES OF REAL NUMBERS..... 156
4.2.1 Introduction..... 156
4.2.2 Series of nonnegative terms..... 160
4.2.3 Absolute convergence..... 164

4.3 SEQUENCES OF K-TUPLES OF REAL NUMBERS..... 165
4.3.1 Convergent sequences..... 165
4.3.2 Subsequences..... 168

CHAPTER 5 LIMITS AND CONTINUITY..... 172

5.1 LIMITS OF REAL FUNCTIONS OF ONE REAL VARIABLE..... 172
5.1.1 Introduction..... 172
5.1.2 Limits..... 182
5.1.3 Operations on limits..... 207
5.1.4 Upper and lower limits..... 217
5.1.5 Other properties of limits..... 229

5.2 LIMITS OF REAL FUNCTIONS OF SEVERAL REAL VARIABLES..... 239
5.2.1 Introduction..... 239
5.2.2 Limits..... 243

5.3 CONTINUITY OF REAL FUNCTIONS OF ONE REAL VARIABLE..... 254
5.3.1 Continuous functions..... 254
5.3.2 Uniform continuity..... 263

5.4 CONTINUITY OF REAL FUNCTIONS OF SEVERAL REAL VARIABLES 265

5.4.1 Continuous functions……..…..……. 265

CHAPTER 6 DIFFERENTIATION..... 273

6.1 DIFFERENTIATION OF REAL FUNCTIONS OF ONE REAL VARIABLE…….....
273
6.1.1 The derivative of a real function.....…… 273
6.1.2 Mean value theorems……...…….…….... 293
6.1.3 Local extrema…….. 301
6.1.4 Taylor's formula…...…..…….. 310
6.1.5 L'Hospital's rule…….. 318

6.2 DIFFERENTIATION OF REAL FUNCTIONS OF SEVERAL REAL VARIABLES...……..…..…….. 324
6.2.1 The partial derivative of a real function..…… 324
6.2.2 Differentiable functions..…….. 332
6.2.3 Derivative of the composite function…… 339
6.2.4 Directional derivative……... 346
6.2.5 Local extrema……..……. 352
6.2.6 Implicit functions..…… 364