CHAPTER 3

KINEMATICS *

3.1 Kinematics of a structure

3.1.1 Rigid bodies kinematics

We already have detailed that we consider plane structures constituted by rectilinear beams, all connected among them and the foundation by constraints. As obvious, to analyze the structure it is fundamental first of all to understand if, supposing rigid every beam and the foundation, the constraints are sufficient to prevent any beam movement or not. So we begin the study of the *kinematics* of the structure supposing rigid its beams.

Let us denote with $\mathfrak{C}_1, \ldots, \mathfrak{C}_t \quad (t \in \boldsymbol{N})$ the beams of the structure. Evidently $\forall i \in \{1, \ldots, t\}$ the beam $\mathfrak{C}_i$, being a rigid body, can change its position in the plane α of the structure only by a translation and/or a rotation, if the constraints allow the movement. Precisely, let us suppose that

- O, x, y be a *Cartesian* orthogonal reference frame (fig. 3.1.1)
- $\mathfrak{C}_i$ moves him in a new position $\mathfrak{C}_i'$ allowed by constraints
- $P_i = \left(x_{P_i}, y_{P_i}\right)$ [resp. Q_i] is the first [resp. second] end of $\mathfrak{C}_i$
- $dist(P_i, Q_i) \in]0, +\infty[$
- the kinematic mechanism happens in the field of small displacements.

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Evidently we can put the segment $\mathfrak{C}_i$ in the new position $\mathfrak{C}_i'$ in two step (fig. 3.1.1)

- in the first step we translate $\mathfrak{C}_i$ of $\boldsymbol{s_i} = (u_i, v_i)$, where u_i and v_i are real numbers in modulus near to zero. In this step P_i [resp. Q_i] moves in the new position $P_i' = \left(x_{P_i} + u_i, y_{P_i} + v_i\right)$ [resp. Q_i']
- in the second step we rotate the segment $P_i'Q_i'$ of φ_i radians, where φ_i is a real number in modulus near to zero. In this step $\mathfrak{C}_i$ gains the final position $\mathfrak{C}_i'$.

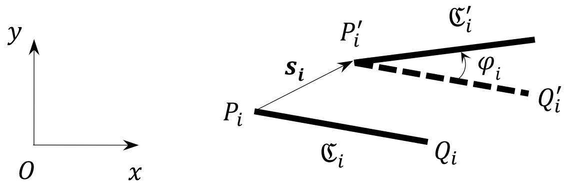

Fig. 3.1.1

Remark 3.1.1 Obviously if $\boldsymbol{s_i} = 0$ the kinematic mechanism is simply a rotation around P_i. If $\varphi_i = 0$ the kinematic mechanism is simply a translation. □

Nevertheless we can place the segment $\mathfrak{C}_i$ in the new position $\mathfrak{C}_i'$ also in three step (fig. 3.1.2)

- in the first step we translate $\mathfrak{C}_i$ of $\bar{\boldsymbol{s}}_{\boldsymbol{i}} = (u_i, 0)$, where u_i is a real number in modulus near to zero. In this step P_i [resp. Q_i] moves in the new position $\bar{P}_i = \left(x_{P_i} + u_i, y_{P_i}\right)$ [resp. $\bar{Q}_i$]
- in the second step we translate the segment $\bar{P}_i\bar{Q}_i$ of $\bar{\bar{\boldsymbol{s}}}_{\boldsymbol{i}} = (0, v_i)$, where v_i is a real number in modulus near to zero. In this step $\bar{P}_i$ [resp. $\bar{Q}_i$]] moves in the new position $P_i' = \left(x_{P_i} + u_i, y_{P_i} + v_i\right)$ [resp. Q_i']
- in the third step we rotate the segment $P_i'Q_i'$ of φ_i radians, where φ_i is a real number in modulus near to zero. In this step $\mathfrak{C}_i$ gains the final position $\mathfrak{C}_i'$.

Such way $\forall i \in \{1, \dots, t\}$ the position of the rigid beam $\mathfrak{C}_i$ in the plane α of the structure is individualized by an ordered triplet u_i, v_i, φ_i of real numbers, small in modulus by the hypothesis of small displacements. The real numbers u_i, v_i individualize the new position of any point P_i of $\mathfrak{C}_i$. The real number φ_i individualizes the rotation of $\mathfrak{C}_i$.

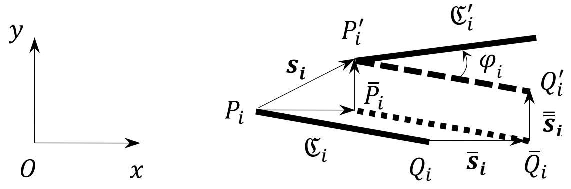

Fig. 3.1.2

As a consequence, choosing $\forall i \in \{1, \dots, t\}$ any point P_i of $\mathfrak{C}_i$, the position of the *t* rigid beams $\mathfrak{C}_1, \dots, \mathfrak{C}_t$ of the structure is individualized by the 3*t* parameters

$$u_1, v_1, \varphi_1, \dots, u_t, v_t, \varphi_t. \tag{3.1.1}$$

We call *release degree* or *lability degree* of the structure, and denote with the symbol $\boldsymbol{r}$, the number of the independent parameters (3.1.1). So $\boldsymbol{r}$ is an integer not negative number such that

$$0 \le \boldsymbol{r} \le 3t. \tag{3.1.2}$$

Remark 3.1.2 In the *Civil engineering* the most advisable choice is $\boldsymbol{l} = 0$. In fact the structure correctly works only if none of its beams can freely move. □

Obviously the constraints present in the structure don't allow to arbitrarily choose the 3*t* parameters (3.1.1). Precisely every constraint imposes to the 3*t* parameters (3.1.1) some restrictions that are analytically translated in equations [3.1.1]. Such equations are, in the hypothesis of small displacements, algebraic linear in the parameters (3.1.1). They are called *constraint equations* and their number, that is called *constraint order*, points out the number of release degrees that the constraint *can* eliminate.

REMARK 3.1.3 The constraint equations are a perfect analytic simulation of the constraint. □

Fixed joint

As seen in sec. 2.1.2, the fixed joint connects, making united, two bodies $\mathfrak{C}_i, \mathfrak{C}_j$ of the structure. It can eliminate three release degrees and then it is a constraint of order 3. Evidently, choosing $P_i = P_j$, the internal fixed joint is simulated by the three constraint equations

$$(3.1.3) \qquad \begin{aligned} u_i &= u_j \\ v_i &= v_j \\ \varphi_i &= \varphi_j \end{aligned}$$

and the external fixed joint is simulated by the three constraint equations

$$(3.1.4) \qquad \begin{aligned} u_i &= 0 \\ v_i &= 0 \\ \varphi_i &= 0\,. \end{aligned}$$

We notice that the (3.1.3), (3.1.4) are algebraic linear equations in the real parameters (3.1.1).

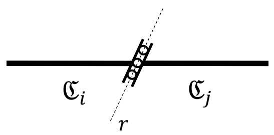

Fig. 3.1.3

[3.1.1] The constraints that are mathematically represented by equations [resp. inequalities] are called *bilateral* [resp. *unilateral*]. In this book we only consider bilateral constraints. In presence of unilateral constraints the analytical treatment is more complex because the employment of the *Functional analysis* is mandatory.

Sliding joint

As seen in sec. 2.1.2, the sliding joint connects two bodies $\mathfrak{C}_i, \mathfrak{C}_j$ of the structure only allowing relative translations according a fixed direction r (fig. 3.1.3). It can eliminate two release degrees and then it is a constraint of order 2. Precisely let (fig. 3.1.4)

- P_i [resp. P_j] a point of $\mathfrak{C}_i$ [resp. $\mathfrak{C}_j$] such that $P_i = P_j$
- n_x, n_y be the direction cosines of a straight line n orthogonal to r.

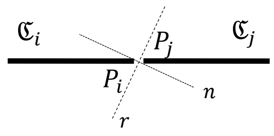

Fig. 3.1.4

Evidently the internal sliding joint is simulated by the two constraint equations

$$\boldsymbol{s_i} \times \boldsymbol{n} = \boldsymbol{s_j} \times \boldsymbol{n}$$
$$\varphi_i = \varphi_j$$

that is

$$u_i n_x + v_i n_y = u_j n_x + v_j n_y$$
(3.1.5)
$$\varphi_i = \varphi_j \ .$$

Analogously the external sliding joint is simulated by the two constraint equations

$$u_i n_x + v_i n_y = 0$$
(3.1.6)
$$\varphi_i = 0 \ .$$

We notice that the (3.1.5), (3.1.6) are algebraic linear equations in the real parameters (3.1.1).

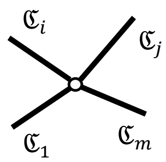

Fig. 3.1.5

Hinge

As seen in sec. 2.1.2, the hinge connects, only allowing relative rotations around the center of the hinge, $m \in \boldsymbol{N}$ bodies $\mathfrak{C}_1, \dots, \mathfrak{C}_m$ of the structure (fig. 3.1.5). One of the bodies $\mathfrak{C}_1, \dots, \mathfrak{C}_m$ can be the foundation.

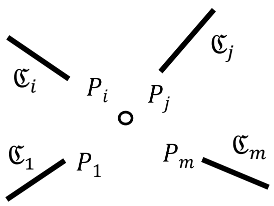

Fig. 3.1.6

The hinge can eliminate $2(m-1)$ release degrees and then it is a constraint of order $2(m-1)$. Precisely $\forall i \in \{1, \dots, m\}$ let P_i be a point of the body $\mathfrak{C}_i$ such that $\forall i, j \in \{1, \dots, m\}$ $\quad P_i = P_j$ (fig. 3.1.6). Evidently the internal hinge is simulated by the $2(m-1)$ constraint equations

(3.1.7)
$$u_2 = u_1, \dots, u_m = u_1$$
$$v_2 = v_1, \dots, \quad v_m = v_1.$$

If the body $\mathfrak{C}_1$ is the foundation, the external hinge is simulated by the

$2(m-1)$ constraint equations

$$u_2 = 0\,, \dots, u_m = 0 \qquad (3.1.8)$$
$$v_2 = 0, \dots, \quad v_m = 0.$$

We observe that the (3.1.7), (3.1.8) are algebraic linear equations in the real parameters (3.1.1).

Pendulum

As seen in sec. 2.1.2, the pendulum is a constraint that connects, by two hinges inserted in the holes P_i and P_j, two bodies $\mathfrak{C}_i, \mathfrak{C}_j$ of the structure (fig. 3.1.7).

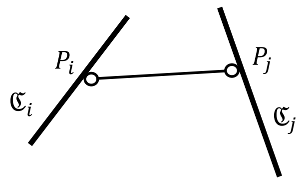

Fig. 3.1.7

The pendulum doesn't allows $dist(P_i, P_j,)$ to vary. So it can eliminate one release degree and then it is a constraint of order 1. Precisely let

- (x_1, y_1) [resp. (x_2, y_2)] be the coordinates of P_i [resp. P_j]
- $P_i' = (x_1 + u_1, y_1 + v_1)$ [resp. $P_j' = (x_2 + u_2, y_2 + v_2)$] the positions reached by P_i [resp. P_j] after a possible kinematic movement of the structure.

Because its rigidity, the pendulum is evidently simulated by the constraint equation

$$dist(P'_i, P'_j) = dist(P_i, P_j),$$

and then

$$f(u_1, v_1, u_2, v_2) = dist^2(P_i', P_j') - dist^2(P_i, P_j) = 0,$$

that is

(3.1.9) $$(x_2 + u_2 - x_1 - u_1)^2 + (y_2 + v_2 - y_1 - v_1)^2 -$$
$$-(x_2 - x_1)^2 - (y_2 - y_1)^2 = 0.$$

Since the hypothesis of small displacements, we can approximate $f(u_1, v_1, u_2, v_2)$ obtaining

(3.1.10) $$f(u_1, v_1, u_2, v_2) = f(0,0,0,0) + \frac{\partial f}{\partial u_1}(0,0,0,0)\, u_1 +$$
$$+\frac{\partial f}{\partial v_1}(0,0,0,0)\, v_1 + \frac{\partial f}{\partial u_2}(0,0,0,0)\, u_2 + + \frac{\partial f}{\partial v_2}(0,0,0,0)\, v_2.$$

From the (3.1.9), (3.1.10) it follows that, in the hypothesis of small displacements, the pendulum is simulated by the constraint equation

(3.1.11) $$-2(x_2 - x_1)u_1 - 2(y_2 - y_1)v_1 +$$
$$+2(x_2 - x_1)u_2 - 2(y_2 - y_1)v_2 = 0$$

algebraic linear in the parameters (3.1.1).

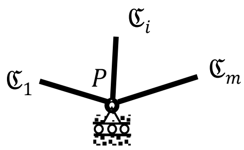

Fig. 3.1.8

Bogie

As seen in sec. 2.1.2, the bogie is a constraint that connects (fig. 3.1.8)

- by one hinge inserted in the hole P, $m \in \boldsymbol{N}$ bodies $\mathfrak{C}_1, \dots, \mathfrak{C}_m$ of the structure

- by two or more wheels, in line and having equal diameter, the hinge with a double track (integral with the rigid foundation) in which the wheels have to roll without attrition.

The bogie *can* eliminate $2m-1$ release degrees, so it is a constraint of order $2m-1$. We consider $\forall i \in \{1,\dots,m\}$ the point P_i of the body $\mathfrak{C}_i$ such that $P_i = P$. Evidently the bogie is simulated by the constraint equations

$$v_1 = 0, \dots, v_m = 0$$

(3.1.12)

$$u_2 = u_1, \dots, u_m = u_1.$$

We notice that the (3.1.12) are $2m-1$ algebraic linear equations in the parameters (3.1.1).

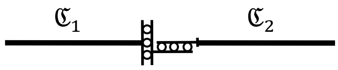

Fig. 3.1.9

Double sliding joint

As seen in sec. 2.1.2, the double sliding joint is a constraint that connects two beams $\mathfrak{C}_1, \mathfrak{C}_2$ of the structure (fig. 3.1.9)

- allowing their relative translation according any direction
- not allowing their relative rotation.

The double sliding joint *can* eliminate one release degree, so it is a constraint of order 1. Evidently the internal double sliding joint is simulated by the constraint equation

(3.1.13) $$\varphi_1 = \varphi_2.$$

If $\mathfrak{C}_1$ is the foundation, the external double sliding joint is simulated by the constraint equation

$$\varphi_2 = 0\,. \tag{3.1.14}$$

We observe that the both (3.1.13) and (3.1.14) is one algebraic linear equation in the parameters (3.1.1).

3.1.2 The kinematic matrix

Let us again consider the structure constituted by the $t \in \boldsymbol{N}$ beams $\mathfrak{C}_1, \dots, \mathfrak{C}_t$. Choosing $\forall i \in \{1, \dots, t\}$ any point P_i of $\mathfrak{C}_i$, the position of the t rigid beams $\mathfrak{C}_1, \dots, \mathfrak{C}_t$ of the structure is individualized by the $3t$ parameters $u_1, v_1, \varphi_1, \dots, u_t, v_t, \varphi_t$, that is by the $3t$ real numbers

$$\begin{gathered} x_1 = u_1 \\ x_2 = v_1 \\ x_3 = \varphi_1 \\ x_4 = u_2 \\ x_5 = v_2 \\ x_6 = \varphi_2 \\ \cdots \\ x_{3t-2} = u_t \\ x_{3t-1} = v_t \\ x_{3t} = \varphi_t . \end{gathered} \tag{3.1.15}$$

Such $3t$ real numbers cannot take arbitrary values, since every constraint requires them some restrictions that are analytically translated, in the hypothesis of small displacements, in the previous homogeneous algebraic linear equations.
So, if we denote with s the sum of the orders of the constraints present in the structure, the parameters (3.1.15) have to satisfy the homogeneous system of algebraic linear equations

$$\begin{gathered} c_{11}x_1 + \cdots + c_{13t}x_{3t} = 0 \\ \cdots \\ c_{s1}x_1 + \cdots + c_{s3t}x_{3t} = 0. \end{gathered} \tag{3.1.16}$$

We call *kinematic matrix* and denote with the symbol C the coefficient matrix of the system (3.1.16), that is the rectangular matrix $s \times 3t$

(3.1.17) $$C = \begin{bmatrix} c_{11} & \cdots & c_{13t} \\ \cdots & \cdots & \cdots \\ c_{s1} & \cdots & c_{s3t} \end{bmatrix}.$$

Let r_C be the rank of C. So r_C is the order of a minor square array I of C of maximum order whose determinant is not zero. Evidently

(3.1.18) $$0 \leq r_C \leq s$$
(3.1.19) $$0 \leq r_C \leq 3t.$$

Let us consider the linear algebraic system B obtained from the system (3.1.16)

- eliminating the $h = s - r_C$ lines that don't belong to I
- translating on the right in the remaining equations (3.1.16), in the column of the known terms, the $3t - r_C$ columns that don't belong to I.

Evidently the linear algebraic system B, having I (whose determinant is not zero) as matrix of the coefficients, is a *Cramer* system. Insofar arbitrarily assigning the $3t - r_C$ parameters we individualize all the parameters (3.1.15), and then the position in the plane of all the beams of the structure. As a consequence, taking into account the (3.1.2), the structure has lability degree $\boldsymbol{r}$ given by

(3.1.20) $$\boldsymbol{r} = 3t - r_C .$$

Obviously the elimination from the system (3.1.16) of the

(3.1.21) $$\boldsymbol{h} = s - r_C$$

lines that don't belong to I it means

- to consider a new structure obtained from that in examination degrading its constraints, only in part or entirely
- that such new structure has the same degree of lability of the

structure in examination.

In other words, in reality the constraints present in the structure remove only r_C of the s release degrees that they are able to eliminate. Insofar in the structure they are present $\boldsymbol{h}$ superabundant constraints, that we call *hyperstatic constraints*. We also use to say that the structure is hyperstatic $\boldsymbol{h}$ times. Taking into account the (3.1.21) we get

(3.1.22) $$0 \leq \boldsymbol{h} \leq s.$$

We say,

- if $\boldsymbol{h} > 0$, that the structure is $\boldsymbol{h}$ *times hyperstatic*
- if $\boldsymbol{h} = 0$, that the constraints are *strictly necessary*
- if $\boldsymbol{r} > 0$, that the structure is $\boldsymbol{r}$ *times labile*
- if $\boldsymbol{r} = 0$, that the structure is *not labile*
- if $\boldsymbol{r} = 0$ and $\boldsymbol{h} = 0$, that the structure is *isostatic*.

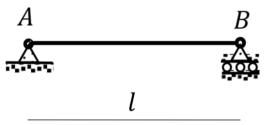

Fig. 3.1.10

We emphasize that the (3.1.20), (3.1.21) imply the *fundamental relation*

(3.1.23) $$3t - s = \boldsymbol{r} - \boldsymbol{h}.$$

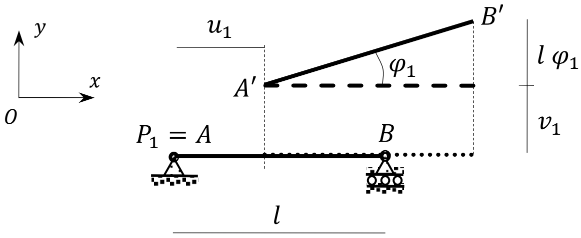

Fig. 3.1.11

PROBLEM 3.1.1 *You shall perform the kinematic analysis of the supported beam of fig. 3.1.10.*

Solution. In the kinematic analysis we must assume rigid the beams of the structure. We observe that the structure is constituted by one beam, so that $t = 1$. We choose the reference frame of fig. 3.1.11 and $P_1 = A$. We consider a new position of the structure, individualized by the three real numbers u_1, v_1, φ_1 (fig. 3.1.11). The constraint equations are evidently

$$\begin{aligned} &u_1 = 0 \\ &v_1 = 0 \\ &v(B) = 0 \end{aligned}$$

so that $s = 3$. Such equations furnish, taking into account the hypothesis of small displacements (fig. 3.1.11)

$$\begin{aligned} &u_1 = 0 \\ &v_1 = 0 \\ &v_1 + l\varphi_1 = 0. \end{aligned}$$

So the kinematic matrix is

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & l \end{bmatrix}.$$

Since $l \in]0, +\infty[$, C has rank 3. As a consequence, from (3.1.20), (3.1.21), we obtain $\boldsymbol{r} = 0$, $\boldsymbol{h} = 0$. So the supported beam of fig. 3.1.10 is isostatic. □

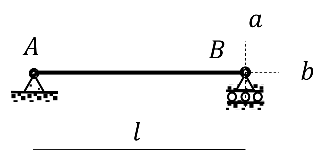

Fig. 3.1.12

REMARK 3.1.4 Evidently in the supported beam of fig. 3.1.12

- the pendulum AB allows B only to move along a (in the field of small displacements)
- the bogie allows B only to move along b (fig. 3.1.4).

So B cannot move and then $\boldsymbol{r} = 0$. From this and from the (3.1.23) we obtain $\boldsymbol{h} = 0$. □

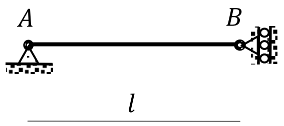

Fig. 3.1.13

PROBLEM 3.1.2 *You shall perform the kinematic analysis of the supported beam of fig. 3.1.13.*

Solution. In the kinematic analysis we must assume rigid the beams of the structure. We observe that the structure is constituted by one beam, so that $t = 1$. We choose the reference frame of fig. 3.1.14 and $P_1 = A$. We consider a new position of the structure, individualized by the three real numbers u_1, v_1, φ_1 (fig. 3.1.14). The constraint equations are evidently

$$\begin{aligned} u_1 &= 0 \\ v_1 &= 0 \\ u(B) &= 0 \end{aligned}$$

so that $s = 3$. Such equations furnish, taking into account the hypothesis of small displacements (fig. 3.1.14)

$$\begin{aligned} u_1 &= 0 \\ v_1 &= 0 \\ u_1 &= 0. \end{aligned}$$

So the kinematic matrix is

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

Clearly C has rank 2. As a consequence, from (3.1.20), (3.1.21), we obtain $\boldsymbol{r} = 1$, $\boldsymbol{h} = 1$. *We point out that the supported beam of fig. 3.1.13 has wrongly arranged constraints and* *then is labile.* □

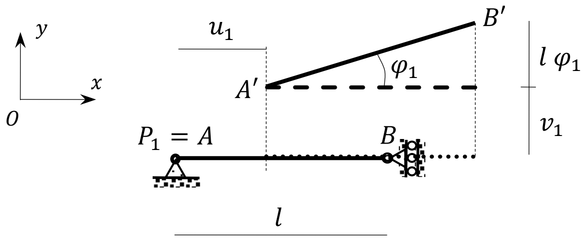

Fig. 3.1.14

Remark 3.1.5 Evidently (fig. 3.1.15) in the structure of fig. 3.1.13

- the pendulum AB allows B only to move along a (in the field of small displacements)
- the bogie allows B only to move along a.

So B can move (along a) and then $\boldsymbol{r} = 1$. From this and the (3.1.23) we obtain $\boldsymbol{h} = 1$. □

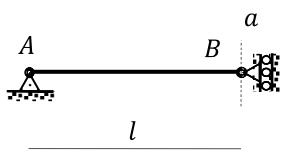

Fig. 3.1.15

Problem 3.1.3 *You shall perform the kinematic analysis of the structure of fig. 3.1.16, where* $0 < \alpha < \frac{\pi}{2}$.

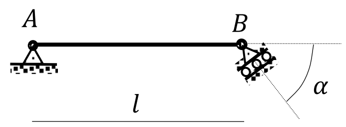

Fig. 3.1.16

Solution. In the kinematic analysis we must assume rigid the beams of the structure. We observe that the structure is constituted by one beam ($t = 1$), and choose the reference frame of fig. 3.1.17 and $P_1 = A$.

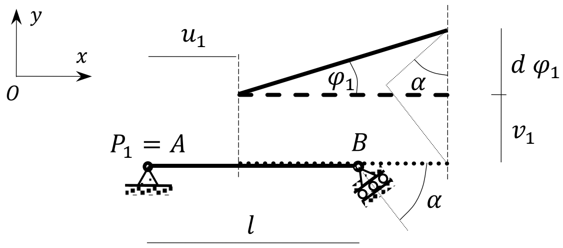

Fig. 3.1.17

We consider a new position of the structure, individualized by the three small real numbers u_1, v_1, φ_1 (fig. 3.1.17). The constraint equations are evidently (fig. 3.1.17)

$$u_1 = 0$$
$$v_1 = 0$$
$$(v_1 + d\varphi_1) \sin \alpha = 0$$

so that $s = 3$. So the kinematic matrix is

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \sin \alpha & l \sin \alpha \end{bmatrix}.$$

Since $0 < \alpha < \frac{\pi}{2}$ and $l < 0$ we have $l \sin \alpha \neq 0$. So C has rank 3. As a consequence, from (3.1.20), (3.1.21), we obtain $\boldsymbol{r} = 0,\ \boldsymbol{h} = 0$. □

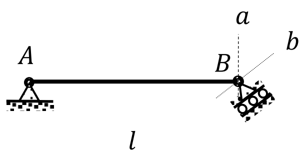

Fig. 3.1.18

Remark 3.1.6 Evidently (fig. 3.1.18) in the structure of fig. 3.1.16

- the pendulum AB allows B only to move along a (in the field of small displacements)
- the bogie allows B only to move along b.

Since $0 < \alpha < \frac{\pi}{2}$ we have $a \neq b$ and then B cannot move. So $\boldsymbol{r} = 0$. From this and (3.1.23) we obtain $\boldsymbol{h} = 0$. □

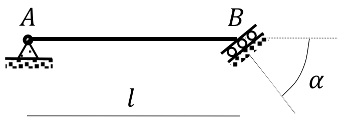

Fig. 3.1.19

Problem 3.1.4 *You shall perform the kinematic analysis of the structure of fig. 3.1.19, where* $0 \leq \alpha \leq \frac{\pi}{2}$.

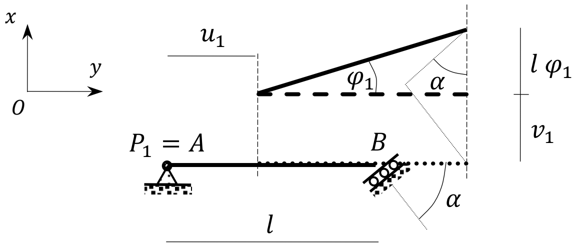

Fig. 3.1.20

Solution. In the kinematic analysis we must assume rigid the beams of the structure. We observe that the structure is constituted by one beam ($t = 1$), and choose the reference frame of fig. 3.1.20 and $P_1 = A$. We consider a new position of the structure, individualized by the three small real numbers u_1, v_1, φ_1 (fig. 3.1.20). The constraint equations are evidently (fig. 3.1.20)

$$u_1 = 0$$
$$v_1 = 0$$

$$(v_1 + l\varphi_1)\sin\alpha = 0$$
$$\varphi_1 = 0$$

and then $s = 4$. So the kinematic matrix is

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \sin\alpha & l\sin\alpha \\ 0 & 0 & 1 \end{bmatrix}.$$

Obviously C has rank 3. As a consequence, from (3.1.20), (3.1.21), we obtain $\boldsymbol{r} = 0$, $\boldsymbol{h} = 1$. □

Remark 3.1.7 Evidently in the structure of fig. 3.1.19

- the hinge A allows the rigid beam AB only to rotate around A
- the sliding joint B doesn't allows the rigid beam AB to rotate.

So $\boldsymbol{r} = 0$. From this and (3.1.23) we obtain $\boldsymbol{h} = 1$. □

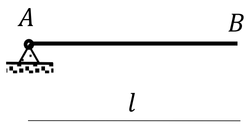

Fig. 3.1.21

Problem 3.1.5 *You shall perform the kinematic analysis of the structure of fig. 3.1.21.*

Solution. In the kinematic analysis we must assume rigid the beams of the structure. We observe that the structure is constituted by one beam, so $t = 1$. We choose the reference frame of fig. 3.1.22 and $P_1 = A$. We consider a new position of the structure, individualized by the three small real numbers u_1, v_1, φ_1 (fig. 3.1.22). The constraint equations are evidently

$$u_1 = 0$$
$$v_1 = 0$$

so that $s = 2$ and the kinematic matrix is

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

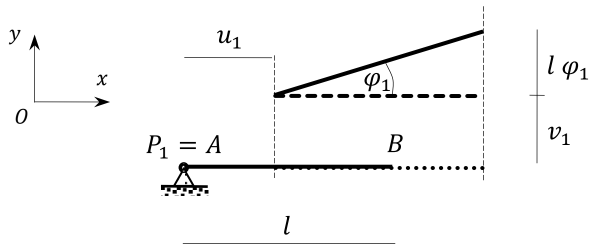

Fig. 3.1.22

Obviously C has rank 2. As a consequence, from (3.1.20), (3.1.21), we obtain $\boldsymbol{r} = 1,\ \boldsymbol{h} = 0$. □

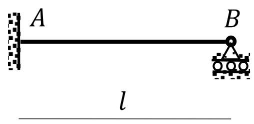

Fig. 3.1.23

REMARK 3.1.8 Evidently in the structure of fig. 3.1.21 the rigid beam AB only can rotate around A. So $\boldsymbol{r} = 1$. From this and from the (3.1.23) we obtain $\boldsymbol{h} = 0$. □

PROBLEM 3.1.6 *You shall perform the kinematic analysis of the structure of fig. 3.1.23.*

Solution. In the kinematic analysis we must assume rigid the beams of the structure. We observe that the structure is constituted by one beam so $t = 1$. We choose the reference frame of fig. 3.1.24 and $P_1 = A$. We consider a new position of the structure, individualized by the three small real numbers u_1, v_1, φ_1 (fig. 3.1.24).

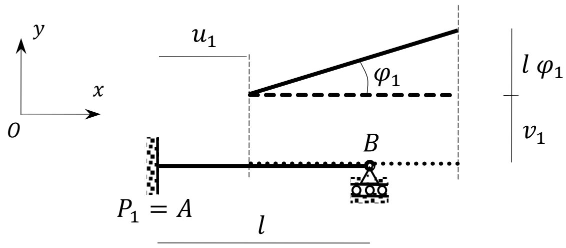

Fig. 3.1.24

The constraint equations are evidently

$$\begin{aligned} u_1 &= 0 \\ v_1 &= 0 \\ \varphi_1 &= 0 \\ v(B) &= 0 \end{aligned}$$

and then $s = 4$. Such equations furnish (fig. 3.1.24)

$$\begin{aligned} u_1 &= 0 \\ v_1 &= 0 \\ \varphi_1 &= 0 \\ v_1 + l\varphi_1 &= 0. \end{aligned}$$

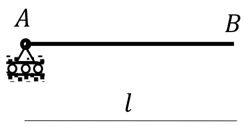

Fig. 3.1.25

So the kinematic matrix is

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & l \end{bmatrix}.$$

Obviously C has rank 3. As a consequence, from (3.1.20), (3.1.21), we obtain $\boldsymbol{r} = 0,\ \boldsymbol{h} =$