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The study of the systems of linear equations consists of four sections.

In the first section, we define the *matrices* and study their properties. Preliminary, we introduce the *vector* (or *linear*) *space* X *over a field* K, by providing X with a suitable binary operation, called *addition*, and with a suitable operation called *scalar multiplication*. Such operations must obey two *distributive laws*. The elements of X are called *vectors* (or *points*), while the elements of K are called *scalars*. If $K = \mathbb{R}$, X is called *real vector* (or *linear*) *space.* A very important example of vector space over a field is the *vector space* $\mathbb{R}^n$ *over the real field*. For the *vector space* $\mathbb{R}^n$ *over the real field* we define:

- the *linear combination* of vectors $\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_k} \in \mathbb{R}^n$ according *coefficients* $c_1, c_2, \dots, c_k \in \mathbb{R}$
- vectors $\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_k} \in \mathbb{R}^n$ *linearly independent*
- vectors $\boldsymbol{x_1}, \boldsymbol{x_2}, \dots, \boldsymbol{x_k} \in \mathbb{R}^n$ *linearly dependent*

and underline their properties. Moreover, we define the *dimension* and the *basis* of such real vector space.

After that, in the first section we define and underline the properties of

- the *rectangular real matrix* (of *dimension* $m \times n$)
- the *square real matrix of order* n
- some particular matrix (*column matrix, row matrix, diagonal matrix, unit matrix, upper triangular matrix, lower triangular matrix, transpose matrix, symmetric matrix, skew-symmetric matrix)*
- the *sum* (or *addition*) of two matrices
- the *product* (or *multiplication*) of a matrix by a real number
- the *product* (or *multiplication*) of two rectangular matrices
- the *invertible* matrix and the *inverse* matrix

In the second section, we define the *determinants* and study their properties. Preliminary, we introduce the c*ombinatorial analysis.* Let $n \in \mathbb{N}$, $k \in \{1, \dots, n\}$, A be the set constituted by n distinct elements

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$a_1, a_2, \dots, a_n$. We define and underline the properties of

- the *k-combination* of the set A, the *k-disposition* of the set A, the *permutation* of the set A
- the *inversion* (both *even* and *odd*) of a permutation.

After that, in the second section we define and underline (also with examples) the properties of

- the *determinant* of order *n,* the determinant of a *singular* matrix, the determinant of a *non-singular* matrix, the rule of *Sarrus,* the *minor* of a determinant of order n, the *complementary minor,* the *algebraic complement* of a minor, the *rank* of a rectangular matrix, the *rank* of the transpose matrix, the *adjoint matrix* of a square matrix.

Moreover, we study

- the determinant of the transpose matrix, the sufficient conditions to annul a determinant, the determinant of the matrix sum, the determinant of the matrix product, the *Rule of Laplace* to calculate a determinant, the *linear dependence* (or *independence*) of the *rows* (or *columns*) of a square matrix.

In the third section, we define and study the properties of

- the *systems of linear equations,* the *matrix of coefficients* of the system, the *column of the known terms,* the *column of the unknowns,* the *augmented matrix,* the *consistent* (or *inconsistent*) system of linear equations, the *homogeneous* system of linear equations.

Moreover, we demonstrate, to solve a system of linear equations

- the *rule of Cramer,* the theorem of *Rouché – Capelli,* the *elimination procedure of Gauss.*

In the fourth section, we present another important problem of the matrix theory: the *eigenvalue problem*. Such problem occurs in various problems of *Geometry, Mechanics, Astronomy, Physics*. We define and study the properties of the *eigenvalues* of a square matrix, the *characteristic equation* (also called *secular equation*) of a square matrix, the *characteristic polynomial* of a square matrix, the *eigenvectors* of a square matrix.